

## A Note on $sg^*$ Closed Sets and their Separation Axioms

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**Abstract:** The aim of this paper is to introduce  $sg^*$  closed set in a Soft topological space and to study some of its properties. The concept  $sg^*$  closure and interior operators are introduced. Then  $sg^*$  soft open and closed sets and soft regular , soft normal sets are derived. Also their properties are derived. .

**Key-Words:**  $sg^*$  open set,  $sg^*$  closed sets,  $sg^*$  soft regular and  $sg^*$  soft normal spaces

**Subject Classification:** 2010MSC: 54C08,54C10

### 1. INTRODUCTION

Molodtsov (1999) introduced the concept of soft set theory. Maji et.al. (2003) initiated the theoretical study on soft sets and investigated some of their properties. After that many authors Maji & Ray (2002), Pei & Miao(2005), Cheng-Fu Yang (2008), Kong et. Al. (2008), Zou & Xiao (2008), Ge & Yang (2011) and Pei Wang & Jiali He (2015) studied the notion of soft sets and applied in many fields of science and engineering. Shabir & Naz (2011) and Cangmen et.al. (2011) initiated the concept of soft topology and investigated some of their properties. Later on, Min (2011), Hussian & Ahmad (2011) and Aygunoglu & Aygun (2012), Zorlutuna et.al (2012), Pazar Varol et.al. (2012), Nazmul & Samanta (2013), PaZAR Varol & Aygun (2013) and Geortiou & Megarities (2014) studied some of the properties of soft topological spaces. Kannan (2012), Arockiarani & Arokia Lancy(2013), Chen (2013), Mahatma & Das (2014), Kandil et.al. (2014a), Guzel Ergul et.al. (2014), Kannan et.al. (2015), Akdag & Ozkan (2014d) and Yuksel et.al.(2014) generated some work forms of open and closed sets in soft topological spaces called soft  $g$  open sets, semi open soft sets, soft regular open sets, pre open soft sets ( $\alpha$  –open soft set, semi open soft sets,  $\beta$  – open soft sets) soft generalized pre regular closed sets, soft stringly  $g$  closed

sets, soft b-open sets and soft regular generalized closed sets respectively in soft topological spaces and studied some of their basic properties.

Shabir & Naz (2011) introduced and investigated the corresponding separation axioms in soft topological spaces using the elements of  $X$ . Using the notion of soft points and soft elements, Cagman et.al. (2011), Hussian & Ahmad(2015) and Nazmul & Samanta (2014) initiated and investigated the corresponding separation axioms in soft topological spaces respectively. Kandil et.al. (2014b) introduced the concept of soft semi separation axioms using the elements of  $X$ . Akdag & Ozjan (2014c) introduced and studied the concept of soft pre separation axioms in soft topological spaces using the elements of  $X$ .

In this paper, the concept of  $sg^*$  closed set is introduced in a soft topological space  $(X, \tilde{\tau} E)$  and some of its properties are studied in section 2. Further, the concept of  $sg^*$  closure and  $sg^*$  interior operators are introduced and some of the fundamental properties are studied. The notion of  $sg^*T_1$  ( $i = 0, 1/2, 1, 2$ ) spaces are introduced in section 3 and characterized using  $sg^*$  open and  $sg^*$  soft closed sets in a soft topological space  $(X, \tilde{\tau} E)$ . Finally,  $sg^*$  soft regular and  $sg^*$  soft normal spaces are introduced and studied some of their basic soft topological properties in section 4.

## 2. $sg^*$ CLOSED SETS

2.2.1 Definition A soft set  $(A, E)$  is called  $sg^*$  closed in a soft topological space  $(X, \tilde{\tau} E)$  of  $cl(A, E) \cong (U, E)$  whenever  $(A, E) \cong (U, E)$  and  $(U, E)$  is soft  $g$  open in  $\tilde{X}$ .

2.2.1 Let  $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$  and

$\tilde{\tau} = \{\tilde{\emptyset}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E), (A_7, E)\}$  where

$$(A_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}, \quad (A_2, E) = \{(b_1, \{a_2\}), (b_2, X)\}$$

$$(A_3, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_2, a_3\})\}, \quad (A_4, E) = \{(b_1, \{a_1, a_3\}), (b_2, X)\},$$

$$(A_5, E) = \{(b_1, \emptyset)\{b_2, \{a_1\}\}) \quad (A_6, E) = \{(b_1, \emptyset)\{b_2, \{a_2, a_3\}\}) \text{ and}$$

$$(A_7, E) = \{(b_1, \emptyset), (b_2, X)\}.$$

Clearly  $(A, E) = \{(b_1, \{a_1, a_3\})(b_2, \{a_3\})\}$  is  $sg^*$  closed in  $(X, \tilde{\tau} E)$ .

since for  $(A, E)$  there exists a soft  $g$  open set  $(U, E) = \{(b_1, \{a_1, a_3\}), (b_2, \{a_2, a_3\})\}$  such that  $cl(A, E) \cong (U, E)$ .

### 2.1 Theorem

Every soft closed set is  $sg^*$  closed in a soft topological space  $(X, \tilde{\tau} E)$ .

**Proof** follows from 2.2.1. Definition and Theorem 1(3) (Shabir & Naz 2011)

### 2.2 Example

The following example shows that the converse of the above 2.2.1. Theorem need not be true.

Let  $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$  &  $\tilde{\tau} = \{\emptyset, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E)\}$

Where  $(A_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}, (A_2, E) = \{(b_1, \{a_3\}), (b_2, \{a_1, a_2\})\}$

$(A_3, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_2\})\}, (A_4, E) = \{(b_1, X), (b_2, \{a_1, a_2\})\}$ , and

$(A_5, E) = \{(b_1, \emptyset)\{b_2, \{a_1\}\})$  are soft open sets,

Here  $(B, E) = \{(b_1, \{a_1\}), (b_2, \{a_1, a_2\})\}$  and  $(C, E) = \{(b_1, \{a_1\}), (b_2, X)\}$  are  $sg^*$  closed sets but are not soft closed sets.

### 2.3 Theorem

Every  $sg^*$  closed set is soft  $g$  closed in the soft topological space  $(X, \tilde{\tau} E)$ .

**Proof** Let  $(U, E)$  be a  $sg^*$  closed set and let  $(B, E)$  be a soft open set such that  $(U, E) \cong (B, E)$ .

By Theorem 4.2 (Kannan 2012).  $(B, E)$  is soft  $g$  open . Since  $(U, E)$   $sg^*$  closed, therefore  $cl(U, E) \cong (B, E)$ .

### 2.4 Theorem

Let  $\{(U, E)_i: i \in J\}$  be the collection of all  $sg^*$  closed sets in a soft topological space  $(X, \tilde{\tau} E)$ . Then  $\tilde{U}_{i \in J} (U, E)_i$  is also a  $sg^*$  closed set in  $(X, \tilde{\tau} E)$ .

Proof. Let  $\tilde{U}_{i \in J} (U, E)_i \cong (V, E)$  and  $(V, E)$  be the soft  $g$  open set. Since  $(U, E)_i$  is  $sg^*$  closed, then  $cl(U, E)_i \cong (V, E)$  for each  $i \in J$ . Hence by Theorem 1(6) (Shabir& Naz 2011)  $cl\tilde{U}_{i \in J} (U, E)_i \cong \tilde{U}_{i \in J} cl(U, E)_i \cong (V, E)$  .

The following example shows that the intersection of two  $sg^*$  closed sets need not be  $sg^*$  closed in a soft topological space  $(X, \tilde{\tau} E)$ .

### 2.5 Example

If  $(U, E)$  and  $(V, E)$  are two  $sg^*$  closed sets in  $(X, \tilde{\tau} E)$ , then  $(U, E) \tilde{\cap} (V, E)$  need not be a  $sg^*$  closed in  $(X, \tilde{\tau} E)$ .

Let  $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$  and

$\tilde{\tau} = \{\tilde{\emptyset}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E), (A_7, E)\}$  where

$(A_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}, \quad (A_2, E) = \{(b_1, \{a_2\}), (b_2, \{a_1, a_2\})\}$

$(A_3, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_2\})\}, \quad (A_4, E) = \{(b_1, \{a_1, a_3\}), (b_2, X)\},$

$(A_5, E) = \{(b_1, \{a_3\}), (b_2, \{a_1, a_3\})\} (A_6, E) = \{(b_1, \emptyset)\{b_2, \{a_1\}\})\}$

And  $(A_7, E) = \{(b_1, \emptyset)\{b_2, \{a_1, \{a_2\}\}\})\}$

Clearly,  $(U, E) = \{(b_1, \{a_1, a_3\}), (b_2, \{a_1, a_3\})\}$  and  $(V, E) = \{(b_1, X), (b_2, \{a_1, a_3\})\}$  are  $sg^*$  closed sets in  $(X, \tilde{\tau} E)$  but  $(U, E) \tilde{\cap} (V, E) = \{(b_1, \{a_1, a_3\}), (b_2, \{a_2\})\}$  is not a  $sg^*$  closed. Set.

## 2.6 Theorem

If  $(U, E)$  is  $sg^*$  closed in a soft topological space  $(X, \tilde{\tau} E)$  and  $\tilde{\subseteq} (V, E) \tilde{\subseteq} cl(U, E)$  then  $(V, E)$  is  $sg^*$  closed.

**Proof** Let  $(V, E) \tilde{\subseteq} (A, E)$  and  $(A, E)$  be a soft  $g$  open set, Since  $(U, E)$  is a  $sg^*$  closed, hence  $cl(U, E) \tilde{\subseteq} (V, E)$ . Therefore  $cl(V, E) \tilde{\subseteq} cl(U, E) \tilde{\subseteq} (A, E)$ .

## 2.7 Theorem

If a soft subset  $(U, E)$  is  $sg^*$  closed in  $(X, \tilde{\tau} E)$ , then  $cl(U, E) - (U, E)$  does not contain any non empty soft  $g$  closed set.

**Proof.** Let  $(F, E)$  be a soft  $g$  closed set such that  $(F, E) \tilde{\subseteq} cl(U, E) - (U, E)$ . Then  $(F, E) \tilde{\subseteq} \tilde{X} - (U, E)$  implies that  $(U, E) \tilde{\subseteq} \tilde{X} - (F, E)$ . Since  $(U, E)$  is  $sg^*$  closed, then  $cl(U, E) \tilde{\subseteq} \tilde{X} - (F, E)$ . That is  $(F, E) \tilde{\subseteq} \tilde{X} - cl(U, E)$ . Hence  $(F, E) \tilde{\subseteq} cl(U, E) \tilde{\cap} (\tilde{X} - cl(U, E)) = \tilde{\emptyset}$ .

## 2.8 Theorem

In a soft topological space  $(X, \tilde{\tau} E)$ , either  $(A_e^x)$  is soft  $g$  closed or  $\tilde{X} - (A_e^x)$  is  $sg^*$  closed.

### **Proof**

Suppose that  $(A_e^x)$  is not soft  $g$  closed. Then  $\tilde{X} - (A_e^x)$  is not soft  $g$  open set. This implies that  $\tilde{X}$  is the only soft open set containing  $\tilde{X} - (A_e^x)$ . Hence  $\tilde{X} - (A_e^x)$  is a  $sg^*$  closed.

## 3. SEPARATION AXIOMS

**3.1 Definition** A soft topological space  $(X, \tilde{\tau} E)$  is said to  $sg^* T_0$  space if for pair of distinct points  $x, y \in X$ , there exists a  $sg^*$  open set  $(A, E)$  such that either  $x \tilde{\in} (A, E)$  or  $x \tilde{\notin} (A, E)$  and  $y \tilde{\in} (A, E)$ .

### 3.2 Definition

A soft topological space  $(X, \tilde{\tau} E)$  is said to be  $sg^* T_1$  space if for each pair of distinct points  $x, y \in X$ , there exists  $sg^*$  open sets  $(A, E)$  and  $(B, E)$  such that  $x \tilde{\in} (A, E)$  but  $y \tilde{\in} (B, E)$  and  $x \tilde{\in} (B, E)$  but  $y \tilde{\in} (A, E)$ .

### 3.3 Theorem

Let  $(X, \tilde{\tau} E)$  be a soft topological space,  $(U, E)$  be a soft set in  $X$  and  $x \in X$ . Then the following statements hold good.

- i)  $x \tilde{\in} (U, E)$  if and only if  $(x E) \tilde{\subseteq} (U, E)$  ;
- ii) If  $(x E) \tilde{\cap} (U, E) = \tilde{\emptyset}$ , then  $x \tilde{\in} (U, E)$ .

**Proof** (i) and (ii) follows from Definition 13 (Shabir & Naz 2011).

### 3.4 Theorem

Let  $x, y \in X$  be distinct points, If there exist  $sg^*$  open sets  $(A, E)$  and  $(B, E)$  in  $\tilde{X}$  such that  $x \tilde{\in} (A, E)$  and  $y \tilde{\in} \tilde{X} - (A, E)$  and  $y \tilde{\in} (B, E)$  and  $x \tilde{\in} \tilde{X} - (B, E)$ , then the soft topological space  $(X, \tilde{\tau} E)$  is  $sg^* T_\alpha$  and  $(X, T_\alpha)$  is a  $T_\alpha$  space for each  $\alpha \in E$ .

#### Proof

Let  $x, y \in X$  be distinct points and  $(A, E)$  and  $(B, E)$  be  $sg^*$  open sets in  $\tilde{X}$  such that  $x \in (A, E)$  and  $y \tilde{\in} \tilde{X} - (A, E)$  or  $y \tilde{\in} (B, E)$  and  $x \tilde{\in} \tilde{X} - (B, E)$ . Hence  $(X, \tilde{\tau} E)$  is  $sg^* T_\alpha$  space. By proposition 5 (Shabir & Naz 2011) for each  $i \in E, (X, \tau_i)$  is a topological space, hence  $x \in A(i)$  and  $y \notin A(i)$  or  $x \in B(i)$  and  $x \notin B(i)$ . Therefore  $(X, \tau_i)$  is a  $T_\alpha$  space for each  $i \in E$ .

### 3.5 Theorem

Let  $x, y \in X$  be distinct points. If there exists  $sg^*$  open set  $(A, E)$  and  $(B, E)$  in  $\tilde{X}$  such that  $x \tilde{\in} (A, E)$   $y \tilde{\in} \tilde{X} - (A, E)$  and  $y \tilde{\in} \tilde{X} - (A, E)$  and  $y \tilde{\in} (B, E)$  and  $x \tilde{\in} \tilde{X} -$

$(B, E)$ , then the soft topological space  $(X, \tilde{\tau} E)$  is  $sg^* T_1$  and  $(X, \tau_1)$  is  $T_1$  space for each  $i \in E$ .

**Proof** Analogues to the proof of 2.3.2 Theorem.

### 3.6 Theorem

If  $(x, E)$  is  $sg^*$  closed set for each  $x \in X$ , then the soft topological space  $(X, \tilde{\tau} E)$  is  $sg^* T_1$ .

**Proof**

Let for each  $x \in X$ ,  $(x, E)$  be a  $sg^*$  closed set and  $xy \in X$  such that  $x \neq y$ . Then  $\tilde{X} - (x, E)$  is a  $sg^*$  open set such that  $y \in \tilde{X} - (x, E)$  and  $x \notin \tilde{X} - (x, E)$ . Similarly  $\tilde{X} - (y, E)$  is  $sg^*$  open set such that  $x \notin \tilde{X} - (y, E)$  and  $y \in \tilde{X} - (y, E)$ . Hence  $(X, \tilde{\tau} E)$  is a  $sg^* T_1$  space.

**Remark**

If  $(x, E)$  is a soft closed set for each  $x \in X$  then the soft topological space  $(X, \tilde{\tau} E)$  is a  $sg^* T_1$  space.

**Proof** follows From 2.2.1 Theorem and 2.3.4 Theorem.

### 3.7 Theorem

Let  $(X, \tilde{\tau} E)$  be a soft topological space and  $Z$  be a non empty subset of  $X$ . If  $(X, \tilde{\tau} E)$  is  $sg^* T_0$  space then  $(Z, \tilde{\tau}_Z E)$  is  $sg^* T_0$  space.

**Proof**

Let  $x, y \in X$  be distinct points. Then by assumption, there exist  $sg^*$  open sets  $(A, E)$  and  $(B, E)$  in  $\tilde{X}$  such that  $x \in \tilde{A}$  and  $y \notin \tilde{A}$  or  $y \in \tilde{B}$  and  $x \notin \tilde{B}$ . since  $z \in Z$ , implies that  $z \in \tilde{Z}$ . Hence  $z \in \tilde{Z} \cap \tilde{B} = \tilde{B}$ . Since  $x \in \tilde{A}$ , then  $x \in \tilde{B}(e)$  for all  $e \in E$ , or  $x \notin \tilde{B}(e)$  for some  $e \in E$ . If  $x \in \tilde{B}(e)$  for all  $e \in E$ , then  $x \in \tilde{Z} \cap \tilde{B} = \tilde{B}$ . If  $x \notin \tilde{B}(e)$  for some  $e \in E$ , then

$x \notin \tilde{Z} \cap B(e)$ . Hence  $x \notin \tilde{Z} \cap (B, E) = (\tilde{Z} B, E)$ . Similarly, for  $x \in (A, E)$  and  $z \in (A, E)$ ,  $x \notin \tilde{Z} \cap (A, E) = (\tilde{Z} A, E)$  and  $z \notin \tilde{Z} \cap (B, E) = (\tilde{Z} B, E)$ . Hence  $(Z, \tau_{\tilde{Z}} E)$  is a  $sg^* T_0$  space.

#### 4. $sg^*$ REGULAR AND $sg^*$ NORMAL SPACES

##### 4.1 Definition

A soft topological space  $(X, \tau E)$  is said to be  $sg^*$  regular if for each  $sg^*$  closed set  $(A, E)$  and each point  $x \in \tilde{X} - (A, E)$  there exist disjoint  $sg^*$  open sets  $(B_1, E)$  and  $(B_2, E)$  such that  $x \in (B_1, E)$ ,  $(A, E) \subseteq (B_1, E) \cap (B_2, E) = \tilde{\emptyset}$ .

##### 4.2 Theorem

For a soft topological space  $(X, \tau E)$ , the following statements are equivalent.

- i)  $(X, \tau E)$  is  $sg^*$  regular.
- ii) For each  $x \in X$  and each  $(A, E) \in sg^*O(X, x)$ , there exists a  $(B, E) \in sg^*O(X, x)$  such that  $x \in (B, E) \subseteq sg^*cl(B, E) \subseteq (A, E)$ .
- iii) For each  $sg^*$  closed set  $(F, E)$  of  $\tilde{X}$ ,
 
$$(F, E) = \tilde{\cap} \{sg^*cl(B, E) : \subseteq (B, E), (B, E) \in sg^*O(\tilde{X})\}$$
- iv) For each subset  $(U, E)$  of  $\tilde{X}$  and each  $(A, E) \in sg^*O(\tilde{X})$  with  $(U, E) \cap (A, E) \neq \tilde{\emptyset}$ , there exists  $(B, E) \in sg^*O(\tilde{X})$  such that  $(U, E) \cap (B, E) \neq \tilde{\emptyset}$ , and  $sg^*cl(B, E) \subseteq (A, E)$ .
- v) For each non-empty subset  $(U, E)$  of  $\tilde{X}$  and each  $sg^*$  closed set  $(F, E)$  of  $\tilde{X}$  with  $(U, E) \cap (F, E) \neq \tilde{\emptyset}$ , there exists  $(B, E), (W, E) \in sg^*O(\tilde{X})$  such that  $(U, E) \cap (B, E) \neq \tilde{\emptyset}$ ,  $(F, E) \subseteq (W, E)$  and  $(W, E) \cap (B, E) = \tilde{\emptyset}$ .

Proof



(i)  $\rightarrow$  (ii)

Let  $(A, E) \in sg^*O(\tilde{X}, x)$ .

Then  $x \notin \tilde{X} - (A, E)$  and there exists  $(C, E), (B, E) \in sg^*O(\tilde{X})$  such that  $\tilde{X} - (A, E) \subseteq (C, E), x \notin (B, E)$  and  $(C, E) \cap (B, E) \neq \emptyset$ . Therefore

$(B, E) \subseteq \tilde{X} - (C, E)$  and  $x \notin (B, E) \subseteq sg^*cl(B, E) \subseteq (\tilde{X} - (C, E)) \subseteq (A, E)$ .

(ii)  $\rightarrow$  (iii)

Let  $(\tilde{X} - (F, E)) \in sg^*O(\tilde{X}, x)$ .

Then by (ii) there exists  $(A, E) \in sg^*O(\tilde{X}, x)$  such that

$x \in (A, E) \subseteq sg^*cl(A, E) \subseteq (\tilde{X} - (F, E))$ . Hence  $(F, E) \subseteq \tilde{X} - sg^*cl(A, E)$ .

By taking  $\tilde{X} - sg^*cl(A, E) = (B, E)$ , then  $(B, E) \in sg^*O(\tilde{X})$  and  $(U, E) \cap (B, E) \neq \emptyset$ . Then  $\tilde{X} - (A, E)$  is the  $sg^*$  closed set containing  $(B, E)$ .

Therefore  $sg^*cl(B, E) \subseteq \tilde{X} - (A, E)$ . Hence  $x \notin sg^*cl(B, E)$ .

Therefore,  $(F, E) = \bigcap \{sg^*cl(B, E) : (F, E) \subseteq (B, E), (B, E) \in sg^*O(\tilde{X})\}$

(iii)  $\rightarrow$  (iv)

Let  $(A, E) \in sg^*O(\tilde{X})$  with  $x \in (A, E) \cap (U, E)$ . Then  $x \notin \tilde{X} - (A, E)$  and hence by (iii), there exists a  $sg^*$  open set  $(W, E)$  such that  $\tilde{X} - (A, E) \subseteq (W, E)$  and  $x \in sg^*cl(W, E)$ . By taking  $(B, E) = \tilde{X} - sg^*cl(W, E)$ , then  $(B, E)$  is a  $sg^*$  open set and hence  $(B, E) \cap (U, E) \neq \emptyset$ . Hence  $(B, E) \subseteq (\tilde{X} - (W, E))$  and  $sg^*cl(B, E) \subseteq \tilde{X} - (W, E) \subseteq (A, E)$ .

(iv)  $\rightarrow$  (v)

Let  $(F, E)$  be a  $sg^*$  closed set. Then  $(\tilde{X} - (F, E))$  is a  $sg^*$  open and  $(\tilde{X} - (F, E)) \cap (U, E) \neq \emptyset$ . Then there exists  $(B, E) \in sg^*O(\tilde{X})$  such that

$(U, E) \tilde{\cap} (B, E) \neq \tilde{\emptyset}$  and  $sg^* cl(B, E) \cong \tilde{X} - (F, E)$ . Then by considering  $(W, E) = \tilde{X} - sg^* cl(B, E)$  then  $(F, E) \cong (W, E)$  and  $(W, E) \tilde{\cap} (B, E) \neq \tilde{\emptyset}$ .

(v)  $\rightarrow$  (i)

Let  $(F, E)$  be a  $sg^*$  closed set and  $x \in \tilde{X} - (F, E)$ , then by (v) there exists  $(W, E), (B, E) \in sg^* O(\tilde{X})$  such that  $(F, E) \cong (W, E)$  and  $x \in (B, E)$  and  $(W, E) \tilde{\cap} (B, E) \neq \tilde{\emptyset}$

### 4.3 Theorem

Let  $(X, \tilde{\tau} E)$  be a sot topological space and  $x \in X$ . If  $(X, \tilde{\tau} E)$  is  $sg^*$  regular space then, i)  $x \in (U, E)$  if and only if  $(x, E) \tilde{\cap} (U, E) \neq \tilde{\emptyset}$  for every  $sg^*$  closed set  $(U, E)$ .

ii)  $x \notin (V, E)$  if and only if  $(x, E) \tilde{\cap} (V, E) \neq \tilde{\emptyset}$  for every  $sg^*$  open set  $(V, E)$ .

#### Proof

(i) Given  $(U, E)$  is a  $sg^*$  closed set such that  $x \notin (U, E)$ . Then there exist  $sg^*$  open sets  $(A, E)$  and  $(B, E)$  such that  $x \in (A, E)$  and  $(U, E) \cong (B, E)$ . Hence From 2.3.1 (i) Theorem  $(x, E) \cong (A, E)$ . Hence  $(U, E) \tilde{\cap} (x, E) \neq \tilde{\emptyset}$ .

Conversely, if  $(U, E) \tilde{\cap} (x, E) \neq \tilde{\emptyset}$ , then from 2.3.1 (ii) Theorem,  $x \notin (U, E)$ .

(ii) Given  $(V, E)$  is a  $sg^*$  open set such that  $x \notin (V, E)$ . If  $x \notin U(e)$  for all  $e \in E$ , then  $(x, E) \tilde{\cap} (U, E) = \tilde{\emptyset}$ . If  $x \notin U(e_1)$  and  $x \notin U(e_2)$  for some  $e_1, e_2 \in E$  then  $x \in X - U(e_1)$  and  $x \notin X - U(e_2)$  for some  $e_1, e_2 \in E$ . Hence  $(U, E) \tilde{\cap} (x, E) \neq \tilde{\emptyset}$  and  $\tilde{X} - (U, E)$  is  $sg^*$  closed set such that  $x \notin \tilde{X} - (U, E)$ . Therefore from (i)  $(x, E) \tilde{\cap} (\tilde{X} - (U, E)) = \tilde{\emptyset}$ , hence  $(x, E) \cong (U, E)$  and  $x \in (U, E)$  which is a contradiction. Conversely, if  $(U, E) \tilde{\cap} (x, E) = \tilde{\emptyset}$ , then from 2.3.1 (ii) Theorem  $x \notin (U, E)$ .

#### 4.4 Theorem

A soft topological space  $(X, \tilde{\tau} E)$  is  $sg^*$  regular if and only if for every  $sg^*$  closed set  $(A, E)$  such that  $(x, E) \tilde{\cap} (A, E) = \emptyset$ , there exist disjoint  $sg^*$  open sets  $(A_1, E)$  and  $(A_2, E)$  such that  $(x, E) \tilde{\subseteq} (A_1, E)$ ,  $(A, E) \tilde{\subseteq} (A_2, E)$ .

#### Proof

Suppose that  $(X, \tilde{\tau} E)$  is  $sg^*$  regular space and  $(A, E)$  is a  $sg^*$  closed set such that  $(x, E) \tilde{\cap} (A, E) = \emptyset$ . Then from 2.3.1 (ii) Theorem  $x \notin (A, E)$ .

Hence there exist disjoint  $sg^*$  open sets  $(A_1, E)$  and  $(A_2, E)$  such that  $(x, E) \tilde{\subseteq} (A_1, E)$ ,  $(A, E) \tilde{\subseteq} (A_2, E)$ .

Conversely, if  $(A, E)$  is a  $sg^*$  closed set such that  $(x, E) \tilde{\cap} (A, E) = \emptyset$ , there disjoint  $sg^*$  open set  $(A_1, E)$  and  $(A_2, E)$  and  $(A_1, E) \tilde{\cap} (A_2, E) \neq \emptyset$ .

#### 4.5 Definition

A soft topological space  $(X, \tilde{\tau} E)$  is said to be  $sg^*$  normal if for any pair of disjoint  $sh^*$  closed sets  $(U, E)$  and  $(V, E)$ , there exist disjoint  $sg^*$  open sets  $(A_1, E)$  and  $(A_2, E)$  such that  $(U, E) \tilde{\subseteq} (A_1, E)$  and  $(V, E) \tilde{\subseteq} (A_2, E)$ .

#### 4.6 Theorem

For a soft topological space  $(X, \tilde{\tau} E)$  the following statements are equivalent.

- i)  $(X, \tilde{\tau} E)$  is  $sg^*$  normal
- ii) for any  $sg^*$  closed set  $(U, E)$  and any  $sg^*$  open set  $(A, E)$  containing  $(U, E)$ , there exists a  $sg^*$  open set  $(B, E)$  containing  $(U, E)$  such that

$$sg^* cl (B, E) \tilde{\subseteq} (A, E)$$

**Proof** (i)  $\rightarrow$  (ii) Let  $(A, E)$  be any  $sg^*$  open set containing the  $sg^*$  closed set  $(U, E)$ . Then  $\tilde{X} - (A, E)$  is a  $sg^*$  closed set. By (i) there exist disjoint  $sg^*$  open sets  $(B, E)$  and

$(C, E)$  such that  $(U, E) \cong (B, E)$  and  $\tilde{X} - (A, E) \cong (C, E)$ . Hence  $\tilde{X} - (A, E) \cong sg^*int (C, E)$ . Since  $(B, E) \tilde{\cap} sg^*int (C, E) = \tilde{\emptyset}$ , then  $sg^*cl (B, E) \tilde{\cap} sg^*int (C, E) = \tilde{\emptyset}$  and  $sg^*cl (B, E) = \tilde{X} - sg^*int (C, E) \cong (U, E)$ . Therefore  $(U, E) \cong (B, E) \cong sg^*cl (B, E) \cong (A, E)$ .

(ii)  $\rightarrow$  (i) Let  $(U, E)$  and  $(V, E)$  be any disjoint  $sg^*$  closed set of  $\tilde{X}$ , Since  $\tilde{X} - (V, E)$  is a  $sg^*$  open set containing  $(U, E)$ , there exists a  $sg^*$  open set  $(B, E)$  containing  $(U, E)$  such that  $sg^*cl (B, E) \cong \tilde{X} - (V, E)$

$s (U, E) \cong sg^*int (B, E)$ . By taking  $(A, E) = sg^*int (B, E)$  and  $(C, E) = \tilde{X} - sg^*cl (B, E)$ , then  $(A, E)$  and  $(C, E)$  are disjoint  $sg^*$  open sets such that  $(U, E) \cong (A, E)$  and  $(V, E) \cong (C, E)$ . Therefore  $(X, \tilde{\tau} E)$  is a  $sg^*$  normal.

## References

- [1] D. Molodtsov, Soft set theory-first results, Math, Appl. 37 (1999),(19-31).
- [2] P.K. Maji, R. Biswas and A.R. Roy, Soft set theory, Comput.Math. Appl, 45 (2003),555-562.
- [3] M. Shabir and M.Naz, On soft topological spaces. Comput, Math. Appl, 61 (2011) 1786 -1799.
- [4] A. Aygünoğlu and H.Aygün, Some notes on Soft topological spaces. Neural Comput and Applic., 21 (1) (2012), 113-119
- [5] W.K Min, A note on soft topological spaces, Comput. Math.Appl., 62 (2011), 3524-3528.
- [6] I,Zorlutuna, M. Akdag, W.K. Min and S. Atmaca, Remarks on soft topological spaces, Ann, Fuzzy Math Inform., 3 (2) (2012), 171 – 185.
- [7] S Hussian and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl., 62 (2011) 4058 -4067 .
- [8] B.Pazar Varol and H. Aygün, On soft Hausdorff spaces, Ann. Of Fuzzy Math. Inform., 5 (1) (2013), 15 – 24.
- [9] B.V.S.T. Sai and V.Srinivasa Kumar, On soft semi-separability, Int.Journalof Math. Analysis, 7 (54) (2013), 2663 – 2669.
- [10] N.Levine, Generalized closed sets in topology, Rend. Cric. Mat. Palermo, 19 (2) (1970), 89 -96.
- [11] K Kannan, Soft generalized closed sets in soft topological Spaces, Journal of Theoretical and Appl. Inform. Technology, 37 (1) (2012), 17 -21.
- [12] S. Alkhazaleh, A. R. Saleh and A.N. Hassan, "Soft multi sets theory", Applied Mathematical Sciences, 5 (72) (2011), 3561 – 3573.