A NOTE ON sg* CONTINUOUS MAPPINGS IN SOFT TOPOLOGICAL SPACES

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Abstract: The aim of this paper is to introduce sg*closed set in a Soft topological space and to study some of its properties. Then sg* continuous mapping and irresolute mapping areintroduced and some of its properties are studied. The concept *sg open, sg* closed mappings and *sg**homeomorphism are introduced and their properties are studied.**

Key-Words: sg* continuous mapping , irresolute mapping, *sg homeomorphism**

Subject Classification: 2010MSC: 54C08, 54C10

1 . INTRODUCTION

The theory of soft sets gives a vital mathematical tool for handling uncertainties and vague concepts. In the year 1999, Molodtsov[1] initiated the study of soft sets. Soft set theory has been applied in several directions. Following this Maji, Biswas, and Roy[7,8] discussed soft set theoretical operations and gave an application of soft set theory to a decision making problem. Recently Muhammad Shabir and Munazza Naz introduced the notion of soft topology[10] and established that every soft topology induces a collection of topologies called the parametrized family of topologies induced by the soft topology. Several mathematicians published papers on applications of soft sets and soft topology[1,2,6,11,12,1]. Soft sets and soft topology have applications to data mining, image processing, decision making problems, spatial modeling and neural patterns[3,4,5,7]. In this paper, the concept sg*closed set is introduced in soft topological space and the concept of sg* continuous mapping and *sg** irresolute mapping are introduced and some of their properties are studied. Further the concept *sg** open , sg* closed mappings and *sg**homeomorphism are introduced and some of their basic soft topological properties are investigated. Finally the concept of slightly *sg** continuous mapping is introduced and studied some of its basic concepts.

2. PRELIMINARIES

2.1 Definition A soft set *(A,E)* is called sg* closed in a soft topological space $(X, \tilde{r}\ E)$ of $cl\ (A, E) \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft g open in $\tilde{X}$.

2.2.1 Let $X = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$ and

$\tilde{r} = \{\tilde{\emptyset}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E), (A_7, E)\}$ where

$(A_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\}),$ $\quad (A_2, E) = \{(b_1, \{a_2\}), (b_2, X)\}$

$(A_3, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_2, a_3\})\},$ $\quad (A_4, E) = \{(b_1, \{a_1, a_3\}), (b_2, X)\},$

$(A_5, E) = \{(b_1, \emptyset)\{b_2, \{a_1\})$ $\quad (A_6, E) = \{(b_1, \emptyset)\{b_2, \{a_2, a_3\})$ and

$(A_7, E) = \{(b_1, \emptyset), (b_2, X)\}.$

Clearly $(A, E) == \{(b_1, \{a_1, a_3\})(b_2, \{a_3\})\}$ is sg* closed in $(X, \tilde{r}\ E)$.

since for *(A,E)* there exists a soft g open set $(U, E) = \{(b_1, \{a_1, a_3\}. (b_2, \{a_2, a_3\})$ such that $cl\ (A, E) \tilde{\subseteq} (U, E)$.

2.1 Theorem

Every soft closed set is sg* closed in a soft topological space $(X, \tilde{r}\ E)$.

3. sg* CONTINUOUS MAPPINGS

3.1 Definition

A soft mapping $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ is called sg* continuous if $\mathbf{f}^1\ (U, E)$ is sg* closed in $(X, \tilde{r}, E)$ for every soft closed set *(U,E)* of $(X, \tilde{\omega}, E)$.

3.2. Theorem

Let $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ be a soft mapping from soft topological space $(X, \tilde{r}, E)$ into a soft topological space $(X, \tilde{r}, E)$. Then the following statements are equivalent.

i) $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ is sg* continuous.

ii) The inverse image of each soft open set in $\tilde{Y}$ is sg* open in $\tilde{Y}$.

iii) For each soft subset $(A, E) \tilde{\in} (Y, \tilde{\omega}, E) sg^* cl(\mathbf{f}^{-1}\ (A, E)) \tilde{\subseteq} \mathbf{f}^{-1}\ cl(A, E))$.

i v) For each soft subset $(B, E) \tilde{\in}$ $(X, \tilde{r}, E) \mathfrak{f}(sg^{*}cl\ (B, E)) \tilde{\subseteq}\ cl(\mathfrak{f}(B, E))$.

Proof (i) ⟶ (ii) follows from 3.1 Definition.

(i)⟶(iii)

Let *(A,E)* be a soft subset of $(Y, \tilde{\omega}, E)$. By 3.2.1 Definition $\mathfrak{f}^{-1}\ cl(A, E))$ is a sg* closed set containing $\mathfrak{f}^{-1}\ (A, E)$ and $sg^{*}cl\ (\mathfrak{f}^{-1}\ (A, E))\ \tilde{\subseteq}\ \mathfrak{f}^{-1}\ cl(A, E))$.

(iii) ⟶(iv)

Let $(B, E) \tilde{\in} (Y, \tilde{r}, E)$, then $\mathfrak{f}(B, E) \tilde{\in} (Y, \tilde{\omega}, E)$ Hence from (iii) $sg^{*}cl\ (\mathfrak{f}^{-1}\ (\mathfrak{f}(B, E))\ \tilde{\subseteq}\ \mathfrak{f}^{-1}(\ cl(A, E))$. Therefore $\mathfrak{f}(sg^{*}cl(B, E))\ \tilde{\subseteq}\ cl\mathfrak{f}\ (B, E)$.

(iv) ⟶(i)

Let *(U,E)* be a soft closed set in $\tilde{Y}$. Then by (iv)

$\mathfrak{f}\ (sg^{*}cl\ (\mathfrak{f}^{-1}(U, E)))\ \tilde{\subseteq}\ cl(\mathfrak{f}(\mathfrak{f}^{-1}(U, E))$. Hence $sg^{*}cl\ (\mathfrak{f}^{-1}(U, E)\ \tilde{\subseteq}\ \mathfrak{f}^{-1}(U, E)$. Therefore $\mathfrak{f}^{-1}(U, E)$ is a sg* closed set in $\tilde{X}$.

3.3 Theorem

Let $\mathfrak{f}$: $\tilde{X} \longrightarrow \tilde{Y}$ be a soft continuous mapping from $\tilde{X}$ into $\tilde{Y}$. Then it is sg* continuous.

Proof

(i) ⟶ **(ii)** follows from 3.1 Definition.

(i)⟶(iii)

Let *(A,E)* be a soft subset of $(Y, \tilde{\omega}, E)$. By 3.1 Definition $\mathfrak{f}^{-1}(cl\ (A, E)$ is a sg* closed set containing $\mathfrak{f}^{-1}(A, E)$ and $sg^{*}cl\ (\mathfrak{f}^{-1}(A, E))\ \tilde{\subseteq}\ \mathfrak{f}^{-,\{1}(cl\ (A, E))$.

(iii) ⟶(iv)

Let $(B, E)\ \tilde{\subseteq}\ (X, \tilde{r}, E)$.Then $\mathfrak{f}(B, E) \tilde{\in} (Y, \tilde{\omega}, E)$. Hence from (iii) $sg^{*}cl\ (\mathfrak{f}^{-1}(\mathfrak{f}(B, E)))$ $\tilde{\subseteq}\ \mathfrak{f}^{-1}(cl\mathfrak{f}(B, E))$. Therefore $\mathfrak{f}(sg^{*}cl\ (B, E))\ \tilde{\subseteq}\ cl\mathfrak{f}\ (B, E)$.

(iv) ⟶(i)

Let *(U,E)* be a soft closed set in $\tilde{Y}$. Then by (iv)

$\mathbf{f}(sg^{*}cl(\mathbf{f}^{-1}(U,E))) \tilde{\subseteq} cl(\mathbf{f}(\mathbf{f}^{-1}(U,E))$. Hence $sg^{*}cl(\mathbf{f}^{-1}(U,E) \tilde{\subseteq} \mathbf{f}(U,E)$.

Therefore $\mathbf{f}^{-1}(U,E)$ is a sg* closed set in $\tilde{X}$.

3.4 Theorem

Let $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ be a soft continuous mapping from $\tilde{X}$ into $\tilde{Y}$. Then it is sg* continuous.

Proof

Let *(A,E)* be any soft closed set in $\tilde{Y}$. Then $\mathbf{f}^{-1}(A,E)$ is soft closed in $\tilde{X}$. Therefore by 2.1 Theorem, $\mathbf{f}^{-1}(A,E)$ is sg* closed in $\tilde{X}$.

3.5 Example

The following example shows that the converse of the above 3.2.2 Theorem need not be true.

Let $X = \{a_1, a_2, a_3\}, Y = \{a_1, a_2, a_3\}, E = \{b_1, b_2\}$ and

$\tilde{\tau_1} = \{\tilde{\emptyset}, \tilde{X}, (B_1, E), (B_2, E), (B_3, E), (B_4, E), (B_5, E)\}$

$\tilde{\tau_1} = \{\tilde{\emptyset}, \tilde{X}, (A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E), (A_6, E)\}$ be two soft topological spaces over X and Y respectively. Then (B_1, E), (B_2, E), (B_3, E), (B_4, E), (B_5, E) are soft sets over X and $(A_1, E), (A_2, E), (A_3, E), (A_4, E), (A_5, E)$ are soft sets over Y defined as follows:

$(A_1, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_3\})\}$, $(A_2, E) = \{(b_1, \{a_3\}), (b_2, \{a_1\})\}$,

$(A_3, E) = \{(b_1, \{a_2\}), (b_2, \{a_3\})\}$, $(A_4, E) = \{(b_1, \{a_3\}), (b_2, \emptyset)\}$,

$(A_5, E) = \{(b_1, X), (b_2, \{a_1, a_3\})\}$, $(A_6, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_3\})\}$,

$(B_1, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}$ $(B_2, E) = \{(b_1, \{a_3\}), (b_2, \{a_1, a_3\})\}$,

$(B_3, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_2\})\}$, $(B_4, E) = \{(b_1, X), (b_2, \{a_1, a_2\})\}$,

and $(B_5, E) = \{(b_1, \emptyset), (b_2, \{a_1\})\}$.

Let $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ be a soft mapping defined by $\mathbf{f}(a_1) = a_1$, $\mathbf{f}(a_2) = a_3$, and $\mathbf{f}(a_3) = a_2$. Then $\mathbf{f}$ is sg* continuous map but not soft continuous. Since $f^{-1}(A_1, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_1, a_2,\})\}$,

$f^{-1}(A_2, E) = \{(b_1, \{a_2\}), (b_2, \{a_1\})\}$, $f^{-1}(A_3, E) = \{(b_1, \{a_3\}), (b_2, \{a_2\})\}$,

$f^{-1}(A_4, E) = \{(b_1, \{a_2\}), (b_2, \emptyset)\}$, $f^{-1}(A_5, E) = \{(b_1, X), (b_2, \{a_1, a_2\})\}$,

$f^{-1}(A_6, E) = \{(b_1, \{a_2, a_3\}), (b_2, \{a_2\})$ are sg* open sets in $\widetilde{r_1}$ but

$f^{-1}(A_3, E)$, $f^{-1}(A_4, E)$, $f^{-1}(A_5, E)$, $f^{-1}(A_6, E)$ are not soft open sets in $\widetilde{r_1}$.

3.6 Theorem

If $f: \tilde{X} \longrightarrow \tilde{Y}$ is a sg* continuous mapping from $\tilde{X}$ into $\tilde{Y}$ then f is soft g continuous.

Proof Let (A,E) be any soft closed set in $\tilde{Y}$. Then $f^{-1}(A, E)$ is sg* closed in $\tilde{X}$. Therefore by 2.1 Theorem $f^{-1}(A, E)$is soft g closed in $\tilde{X}$.

3.7 Definition

A soft mapping $f: \tilde{X} \longrightarrow \tilde{Y}$ called sg* irresolute if $f^{-1}(U, E)$ is sg* closed in $\tilde{X}$ for every sg* closed set of $(Y, \widetilde{\omega}, E)$.

3.8 Remark

A soft mapping $f: \tilde{X} \longrightarrow \tilde{Y}$ is sg* irresolute if and only if the inverse image of every sg* open set in $(Y, \widetilde{\omega}, E)$ is sg* open in $\tilde{X}$.

3.9 Theorem If $f: \tilde{X} \longrightarrow \tilde{Y}$ and $h: \tilde{Y} \longrightarrow \tilde{Z}$ are any two soft mappings then

i) $h \circ g$ is sg* continuous if h is soft continuous and f is sg* continuous.

ii) $h \circ g$ is sg* continuous if h is sg* continuous and g is sg* irresolute.

iii) $h \circ g$ is sg* irresolute if both g and h are sg* irresolute.

Proof

(i) Let *(U,E)* be a soft closed set in $\tilde{Z}$. Then $h^{-1}(U, E)$ is soft closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U, E)) = h^{\circ}\, g)(U, E)$ is sg* closed in $\tilde{X}$.

(ii) Let *(U,E)* be a soft closed set in $\tilde{Z}$. Then $h^{-1}(U, E)$ is sg* closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U, E)) = h^{\circ}\, g)(U, E)$ is sg* closed in $\tilde{X}$.

(iii) Let (U,E) be a sg* closed set in $\tilde{Z}$. Then $h^{-1}(U, E)$ is sg* closed in $\tilde{Y}$ and $g^{-1}(h^{-1}(U, E)) = h^{\circ}\, g)(U, E)$ is sg* closed in $\tilde{X}$.

3.10 Theorem

A soft mapping $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ is sg* irresolute if and only if for every soft subset *(U,E)* of $\tilde{X}, g(sg^* \, cl(U,E)) \tilde{\subseteq} sg^* \, cl(g(U,E))$.

Proof Let g be a sg* irresolute mapping and *(U,E)* be a soft subset in $\tilde{X}$. Then $sg^* \, cl(g(U,E))$ is sg* closed set in $\tilde{Y}$. Hence $g^{-1}(sg^* \, cl(g(U,E)))$ is sg* closed in $\tilde{X}$ and $(U,E) \tilde{\subseteq} g^{-1}(g(U,E)) \tilde{\subseteq} g^{-1}(sg^* \, cl(g(U,E)))$.

Therefore

$sg^* \, cl(U,E) \tilde{\subseteq} g^{-1}(sg^* \, cl(g(U,E)))$, hence $g(sg^* \, cl(U,E)) \tilde{\subseteq} g^{-1}(sg^* \, cl(g(U,E))$.

Conversely, suppose that *(U,E)* is sg* closed in $\tilde{Y}$.

Therefore

$g(sg^* \, cl(g^{-1}(U,E))) \tilde{\subseteq} (sg^* \, cl \, (g(g^{-1}(U,E))) = sg^* \, cl(U,E) = (U,E)$. Hence $sg^* \, cl(g^{-1}(U,E)) \tilde{\subseteq} g^{-1}(U,E)$.

4. sg* HOMEOMORPHISMS

4.1 Definition

A soft mapping $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ is called sg* open if $g(U,E)$ of each soft open set *(U,E)* in

$(X, \tilde{r}, E)$ is sg* open in $(Y, \tilde{\omega}, E)$.

4.2 Definition

A soft mapping $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ is called sg* closed if $g(U,E)$ of each soft closed set *(U,E)* in $(X, \tilde{r}, E)$ is sg* closed in $(Y, \tilde{\omega}, E)$.

4.3 Theorem

Let the soft mappings $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ and $g: \tilde{Y} \longrightarrow \tilde{Z}$ be bijective. If $g \circ \boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Z}$ is soft continuous and $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ is soft continuous and $\boldsymbol{f}: \tilde{X} \longrightarrow \tilde{Y}$ is sg* closed then $g: \tilde{Y} \longrightarrow \tilde{Z}$ is sg* continuous.

Proof

Let *(U,E)* be the soft closed set in $\tilde{Z}$. Since $g \circ \mathbf{f}: \tilde{X} \longrightarrow \tilde{Z}$ is soft continuous, then $\mathbf{f}^{-1}(g^{-1}(U,E)) = (g \circ \mathbf{f})^{-1}(U,E)$ is soft closed set in $\tilde{X}$. Since $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ is sg* closed, then $\mathbf{f}(\mathbf{f}^{-1}(g^{-1}(U,E))) = g^{-1}(U,E)$ is sg* closed set in $\tilde{Y}$.

4.5 Theorem

A soft mapping $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ is a sg* open iff if $\mathbf{f}(\mathrm{i}kt(B,U)) \tilde{\subseteq} sg^{*}\mathrm{i}kt\,(\mathbf{f}(B,E))$ for every soft subset *(B,E)* of $\tilde{X}$.

Proof

Let $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ be sg* open and *(B,E)* be a soft subset of $\tilde{X}$, then $\mathrm{i}kt(B,U)$ is a soft open set in $\tilde{X}$. Hence $\mathbf{f}(\mathrm{i}kt(B,E)) = sg^{*}\mathrm{i}kt\,(\mathbf{f}(\mathrm{i}kt(B,E))$.

Conversely, Let *(G,E)* be a soft open set in $\tilde{X}$. $\mathbf{f}(G,E) = \mathbf{f}(\mathrm{i}kt(G,E)) \tilde{\subseteq} sg^{*}\mathrm{i}kt\,(\mathbf{f}(G,E))$, which implies $\mathbf{f}(G,E) \tilde{\subseteq} sg^{*}\mathrm{i}kt\,(\mathbf{f}(G,E))$. Hence $\mathbf{f}(G,E)$ is a sg* open in $\tilde{Y}$.

4.6 Definition

If a soft mapping $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ is sg* continuous bijective and $\mathbf{f}^{-1}$ is sg* continuous then *f* is said to be sg* homeomorphism from $(X, \tilde{r}, E)$ in to $(Y, \tilde{\omega}, E)$.

4.7 Theorem

Let $\mathbf{f}: \tilde{X} \longrightarrow \tilde{Y}$ be the soft bijective mapping. Then the following statements are equivalent: . Since f is sg* open map,

i) $\mathbf{f}^{-1}: \tilde{Y} \longrightarrow \tilde{X}$ is sg* continuous.

ii) f is sg* open .

iii) f is sg* closed.

Proof

(i)⟶(ii) Let *(U,E)* be any soft open set in $\tilde{X}$. Since $\mathbf{f}^{-1}: \tilde{Y} \longrightarrow \tilde{X}$ is sg* continuous, therefore $(\mathbf{f}^{-1})^{-1}(U,E) = \mathbf{f}(U,E)$ is sg* open in $\tilde{Y}$.

(ii) ⟶(iii) Let *(B,E)* be any soft closed set in $\tilde{X}$, then $\tilde{X} - (B, E)$ is soft open set in $\tilde{X}$. Since f is sg* open map, $\mathfrak{f}(\tilde{X} - (B, E))$ is sg* open in $\tilde{Y}$. But $\mathfrak{f}(\tilde{X} - (B, E)) = \tilde{Y} - \mathfrak{f}(B, E)$, implies $\mathfrak{f}(B, E)$ is sg* closed in $\tilde{Y}$.

(iii) ⟶(i) Let *(B,E)* be any soft closed set in $\tilde{X}$. Then $(\mathfrak{f}^{-1})^{-1}(U, E) = \mathfrak{f}(U, E)$ is sg* closed in $\tilde{Y}$. Therefore $\mathfrak{f}^{-1}: \tilde{Y} \longrightarrow \tilde{X}$ is sg* continuous.

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