Exploring grade eleven learners' mathematical modelling competences in problem solving

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ABSTRACT

The teacher is considered as the most crucial factor in implementing all instructional reforms at the grassroots level. Several studies have shown that unqualified teachers have a negative effect on learners' academic performance. The study was motivated by the fact that many teachers did not receive training on mathematical modelling during their pre-service training. So, this study explored the mathematical modelling proficiency of the Grade Eleven learners in three different high schools in Pongola Circuits in KwaZulu-Natal Province. The study determined learners' competency levels in solving non-routine problems. A total of 75 Grade Eleven learners from the three purposively selected schools participated in the study. Qualitative data were gathered through document analysis. Learners' written work, in which they responded to the test questions, was thematically analysed to comprehend meaning, recognise patterns, and obtain insights related to a research question. The results of this investigation revealed that the all learners exhibited incomplete competency in mathematical modelling. The study recommends that mathematics teachers should be urged to use the mathematical modelling approach when instructing and learning the subject. It was inferred that learners were never introduced to mathematical modelling since crucial steps including defining variables, making assumptions, verifying and interpretation of results were omitted by all participants.

KEYWORDS: mathematical competency, constructivism, models, mathematical modelling, modelling processes

INTRODUCTION

Creativity is essential in education. At the school level, the term creative is commonly used in these two subjects: Visual Arts and English Language. In South Africa, there is a section in English language Paper Three, which is referred to as Creative Writing. In this section, learners are required to write a composition and a letter, using their imaginations or original ideas. One of the tangible objectives of the visual art curriculum is to inquire, create and accomplish innovative thoughts in response to projects that are both automatically generated and determined externally by utilising personal expertise and acquainted with visual studies from the historic and current (DBE 2011b). In both subjects, creativity has been emphasised and that is why it is easy for anyone to associate creativity with these two subjects.

Cropley (2020) mentioned that creativity is not restricted to different forms of arts such as visual arts, music, dramatic art, literature and related artistic fields; it also takes place in areas or domains such as technology, business, medicine, manufacturing, defence, administration, as well as in education. Creativity produces concrete material such as books, artworks, music, buildings, machines, and non-tangible outcomes such as ideas, services, processes, systems of operation, to name a few. If all these things are done in a way that is unique and original or effective in obtaining an intended outcome, creativity would have taken place. Creativity may result in quite a number of actions, stretching from those which are abstract, for example, an indication of a feeling, appreciation of beauty and persuasion of a new perspective of looking at an object.

The word creativity may be utilised in three different ways to represent; a variation of practices (for example innovative thinking); a number of individual features of people (for instance the visionary individual); and results (such as an inventive creation). Due to all of these, creativity may be classified as a cause (innovative processes bring by-products) as well as a result (specific type of product manifest from process and person). This is referred to as the 3 Ps technique (person, process and product), that was later developed to include a fourth P (press), representing the influence of the surroundings that can either promote or inhibit innovation (Cropley 2020). As stated in Merriam-Webster Dictionary the term creativity means the capability to create new things or bring up new ideas. Such definition fits well in Mathematics because in Mathematics, learners sometimes have to solve mathematical problems using their own strategies. This is always the case when learners have to resolve non routine problems.

Numerous studies have revealed that when teachers use problem-solving in their Mathematics teaching approach daily, learners' performance in Mathematics improves.

Examples of such studies include those of Albay (2019), Crophey (2020), and Yuan (2013). Also, several studies have shown that mathematical modelling enhances learners' achievement in Mathematics; for example, Sokolowski (2015), Ciltas and Isik (2013), and Nguyen (2016). In fact, mathematical modelling is also a problem-solving method, but the emphasis in modelling is on the use of real-life problems. Hence, this study on mathematical modelling was conducted through algebraic problem-solving. It was hoped that the findings of this study will motivate teachers to start employing mathematical modelling in their classrooms. For this study we posed the research question: *What is the nature of competencies of Grade Eleven learners in mathematical modelling when resolving real world problems*?

LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

The leading philosophical approach to this research is constructivism. I have already given several definitions of constructivism in section Chapter Two. To recap what has been mentioned in those definitions, I found it necessary to look at how other scholars have defined the term constructivism. Constructivism, according to Anyanwu and Iwuamadi (2015), is a method that encourages active participation, problem-solving, and teamwork as learners acquire knowledge based on past experiences. Teachers serve as facilitators in constructivist classrooms, offering guidance and promoting interactive discussions. According to Resnick and Glaser (2016), constructivism is a philosophy of learning or interpretation that allows individuals to generate their own new understanding through the interaction of concepts and knowledge with what they already know and believe. Constructivism, a learning theory that stresses active knowledge building through experience and interaction, was defined by Shittu and Alex (2025). It has become a viable substitute for conventional approaches.

Despite the fact that constructivism started as a theory of learning, it has gradually broadened its dominance a theory that can be used in different areas such as teaching, education, developing ideas, as well as a theory of personal and scientific knowledge. Constructivism has influenced several national curriculum documents and national education statements.

It has been shown that constructivism is no longer limited to learning, which was its main focus. It must be remembered that pioneers of constructivism paid attention on how learners learn and how they construct new ideas using previous learnt experiences. Since constructivism also influence teaching it is not surprising that even the teaching methods have to change, from the traditional approaches that are teacher-centred to those that are learner-centred. Constructivists say that learners construct knowledge when they are actively involved. The influence of constructivism in changes in curriculum has been seen even in South Africa and has resulted in the new curriculum. For example, in Mathematics, a mathematical modelling approach was suggested. This means that teachers also had to change their teaching methods so that they are in line with mathematical modelling approaches.

This chapter indicated that different scholars defined constructivism in different words. However, all these different explanations were saying almost one and the same thing, using different words. They all specify that knowledge is actively created by individuals, who generate new information by building upon their prior knowledge base. Yilmaz (2008) provided the following summary of the fundamental presumptions and tenets of the constructivist theory of learning: 1. Learning is a dynamic process, 2. Learning is a flexible activity, 3. The context of learning determines where it takes place, 4. The learner constructs knowledge; it is neither intrinsic, passively absorbed, or invented, 5. All knowledge is unique and individualized, 6. Knowledge is shaped by society, 7. Learning is basically a process of interpreting the world, 8. Learning is influenced by experienced and past knowledge, 9. Learning is aided by social interaction and 10. Meaningful, demanding, open-ended problems are necessary for effective learning.

Examining some of the above principles thoroughly, one can see that they are in line with the mathematical modelling approaches. These include learning as: an active process; socially constructed; utilisation of open-ended problems; and importance of prior knowledge. Therefore, these made constructivism to be the relevant theoretical framework for this study. The principles of constructivism are also supported by the work of Faulkenberry and Faulkenberry (2006) when they said although there are different forms of constructivism, all have one feature that is common; they put emphasis on learning that is learner-centred. This means during learning in class, learners must be active participants, not just sitting passively waiting for a teacher to transmit knowledge to them. During learning, learners use their own knowledge, prior experiences and their ideas. All these influence how learners are going to acquire new information.

Honebein (1996) cited Cunningham, Duffy and Knuth (1993) as well as Knuth and Cunningham (1993) list seven objectives for teaching constructivist methods. After that, Honebein (1996) gave the following summary of these objectives: 1. To provide experience of the knowledge construction process. This goal means that learners are responsible for deciding the topics they do in their fields of study want to learn and deciding on the learning strategies they will use, 2. To provide the knowledge of and respect for various points of view. Real-life problems have more than one correct strategy or one correct answer. Learners must be involved in activities that allow them to determine or investigate different solutions to a problem, 3. To embed learning in realistic contexts. Problems used in class must identify with problems existing in reality, 4. To inspire the state of owning and of voicing about the learning process. Learners play a major role in deciding what they will learn. This is an indication of learner-centred learning, 5. To embed learning in social experience. This is because cognitive development is influenced by social interacting. Therefore, learning should consider collaboration between teachers and learners as well as among learners themselves, 6. To encourage the presentation of information in various ways. The use of only spoken and written communications will restrict how learners see the world. Other means of communications, for example, computers, photographs and videos must be utilised to provide abundant knowledge and encounters and To foster the perception of the process of acquiring knowledge. Learners' capability to justify their choice of strategy for solving a problem is a crucial end-product of constructivist learning.
Some of the goals associated with teaching through constructivists approach are related to mathematical modelling in that; learning is embedded in real-life contexts, learning is learner-centred, learning activities have more than one correct solutions, learners have the autonomy to use their own procedures in resolving the problems, a learning environment that encourages collaboration, and as a requirement for learners to give full details of how they worked out a particular problem. Therefore, just like the principles of constructivism, the goals associated with teaching using constructivists approach, are strongly linked to mathematical modelling. Hence, this made constructivism to be the relevant theoretical framework for this study which investigated learners' competencies in mathematical modelling.

METHODOLOGY

The study employed the qualitative approach to respond to the research question. The study investigated learners' competencies in mathematical modelling when solving problems involving real-life situations, hence the phenomenological research approach was employed. Creswell (2009) says phenomenological research is employed when a researcher intends to describe a situation and at the same time identify the nature and character of human experience regarding a phenomenon. In this study, we described in written words what was done correctly or incorrectly by Grade Eleven learners that participated in the study when solving real life problems using the mathematical modelling approach.

This study analysed 75 learners' scripts of the test that was designed for this study when they worked in groups. According to Nel (2019), interpretivism pays more attention to humans' personal experience as they create the social world in addition to how they interact with one another.

DATA ANALYSIS AND DISCUSSION

We examined the learners' written work when answering the test that was used to generate data. The learners' scripts were considered as documents. We analysed them using a research technique known as content analysis. Wallen and Fraenkel (2001) define content analysis as the process of analysing textual or visual material that is contained in documents. Furthermore, Patton (2002) regarded content analysis as any qualitative data reduction and sense-making undertaking that takes a large amount of qualitative data and looks for essential consistencies and meanings. According to Cavanagh (1997), content analysis is viewed by researchers as a versatile or flexible technique for examining text data. Gheyle et al (2017) defined content analysis is a method of investigation which make sense of the unorganised content of messages which might be texts, images, symbols or audio data.

Test used in the study

1. Mr Nene wants to plant seedlings in his rectangular garden with an area of 54 m^2. The length of the garden is longer than its width by 3m. What are the dimensions of his garden? (6)

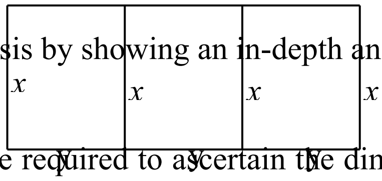
2. To protect his vegetables, a farmer buys 32m of fencing for his rectangular garden. Determine the dimensions of the vegetable garden if it should cover a maximum area (6)

3. A rectangular garden is to be constructed using a rock wall as one side of the garden and wire fencing for the other three sides. Given 72m of wire fencing:

3.1. Determine the dimensions that would create a garden of maximum area. (5)

3.2. What is the maximum area? (1)

4. A farmer has 600m of fence to create a rectangular pasture which has to be divided into three camps (as shown in the sketch below) all with the same dimensions. What is the maximum area that can be enclosed with this fence? (7)



We demonstrate the data analysis by showing an in-depth analysis of question 1. for one question viz question 1.

For this question, learners were required to ascertain the dimensions of a garden with an area of 54m^2 given that the length is longer than the width by 3m. To respond to such a question and show an understanding of the problem, learners were expected to perform these steps:

1. Use a diagram (rectangular drawing) to illustrate the scenario, choose and specify the variables, and assumed that will lead to the correct answer. For instance, to cultivate 54 square meters in this question, there must be no space that is covered with large rocks or boulders; otherwise, even though the dimensions are 9 and 6 meters, the area that is arable will be less than what is needed.
2. Create a correct mathematical model (equation) to depict the scenario by establishing the proper relationships between the chosen variables. Learners were expected to indicate that the variable they had chosen to be the length was

longer than their breadth by 3 metres. Furthermore, they were supposed to indicate that the product of the two dimensions was equal to $54m^2$

3. Obtain accurate mathematical answers by resolving the properly formed equation or mathematical representation (model).
4. Interpret correctly the attained mathematical answer in an actual life setting. This is done by using a complete sentence to respond to the question. For instance, in this question, one of the possible interpretations of results would be "The width of the garden is 6metres and its length is 9metres".
5. Verify and give the correct reason why the solution is considered correct or incorrect and rectify the specified mistake. Checking the answer to an equation or a mathematical problem is a crucial stage since it allows learners to make sure their approach or solution is correct. In this question, learners were supposed to check if one of their obtained answers was longer than the other by 3 metres and their product was 54 square metres.

Prior knowledge from preceding grades in the topics Algebra and Measurement was necessary for the learners to answer every question on the test. In Algebra, they learnt how to solve linear, literal and quadratic equations. Under Measurement, learners learnt about formulae and perimeters of triangles, quadrilaterals, circles and cylindrical surfaces. Since the questions involved rectangular figures, learners needed to know the formula of a perimeter and area of a rectangle. Figure 1 displays how one of the respondents answered Question 1 in the test that was utilised for this investigation. The answer belonged to learner L27, who managed to draw a sketch of the garden, created a relationship between the two variables, *l* and *b*. However, this learner did not specify what each of the two variables represented. There was no assumption made, which is a requisite for mathematical modelling. In the second step, the learner introduced the variable *x* and did not state how this variable related to *l* or *b*. Still on the same step, there was a mathematical error when the learner tried to multiply $x+3$ by x but omitted the brackets in $x+3$. By chance, this learner got the next step correct and proceeded appropriately to get the correct solutions. However, this learner did not select a valid solution and give an explanation why $x=-9$ was an invalid solution in terms of the nature and context of the problem. Lastly, there was no verification of the solutions. Since this learner did not follow all the five steps of the modelling process properly, I described this learner as partially competent in mathematical modelling. In terms of proficiency in mathematical strands, this learner is proficient in strategic ability, conceptual comprehension and procedural proficiency, but not in adaptive thinking. This is because it is clear that the learner understood the problem, developed the correct formula although they omitted the brackets, followed the procedures of solving the problem correctly, but did not explain why -9 was not a valid solution. Also, there was no attempt to validate or verify the solution.

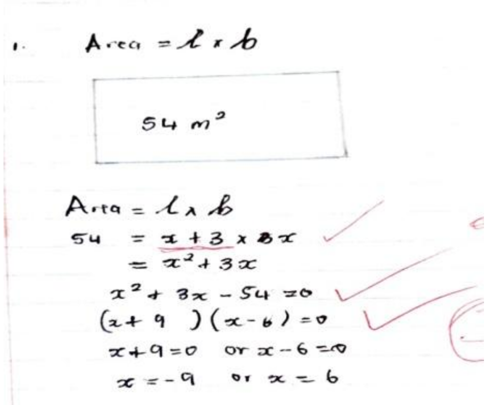


Figure 1: L27 response to Question 1

Figure 2 displays the learner's response where there was no sketch drawn to represent the garden. A statement that should have stated what each of the two variables, *l* and *b*, represented, was missing. One would expect a statement such as "Let *l* represent the length and *b* the breadth". A statement such as this would enable any person to follow the learner's working since she or he would know what the variables represented. Despite the missing diagram and the statement that explain the variables, the learner formulated correct mathematical statements and continued to work correctly. Like the previous learner in Figure 6.3, there was no attempt to assume.

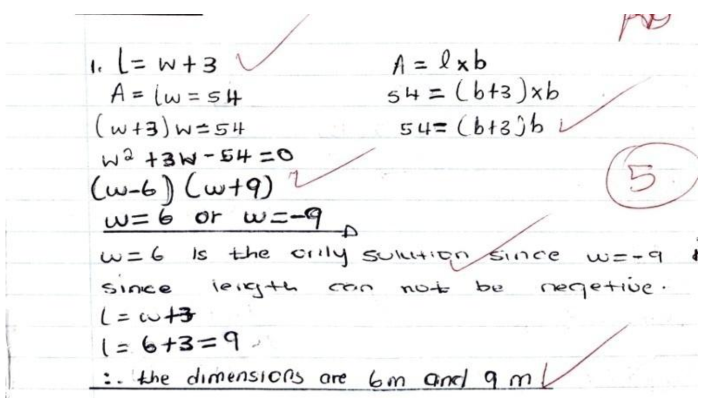


Figure 2: L36 response to Question 1

This learner, L36 in Figure 2 managed to obtain both solutions $b=6$ and $b=-9$ and gave a clear explanation why the solution $b=-9$ was invalid. The learner proceeded and could not get all the marks since they could not calculate the value of the length and validate or verify the solution. Another mistake made by this learner was to use variables *b* and *w* concurrently. If the learner had decided to use *b* for breadth, they would have to change to variable *w* for width. They should have continued working with variable *b* to the end. I regarded learner L36 in Figure 2 as having a strong strategic competence, adaptable thinking, and conceptual comprehension because they managed to develop a correct formula to show that they had understood the problem, solved the equation correctly and justified the solution. However, this learner was partially proficient in procedural fluency, as they could not work out all the solutions. They worked out the breadth, but not the length of the garden.

Figure 3 exhibits a response of a learner who worked carelessly. Note that although the learner managed to develop a correct formula that represented the given information, the learner made a mistake while continuing in solving the equation. They omitted a zero in step 5 of the working and this made the working mathematically incorrect because the statement in step 5 was no longer equal to the one in step 4. By omitting the zero, the learner ended up with the expression $b^2+3b-54$ instead of the equation $b^2+3b-54=0$. However, the learner proceeded and gave the correct answers. The concern was in the explanation of why the solution -9 was invalid. The learner cancelled the correct explanation; consequently, their statement contradicted why the solution *w* = -9 was invalid. Like the other learners, there was no attempt to make an assumption and to verify the solution.

With regard to mathematical proficiency strands, this learner displayed partial proficiency. This is because the learner demonstrated proficiency in conceptual understanding, as they managed to come out with a correct formula and substituted correctly, which indicated that they had understood the problem. Developing the correct formula and continuing to solve it correctly also showed that the learner was proficient in strategic competence and in procedural fluency. However, failing to verify the solutions meant that the learner was not proficient in adaptive reasoning. Therefore, overall, the learner was described as partly proficient in mathematical proficiency strands.

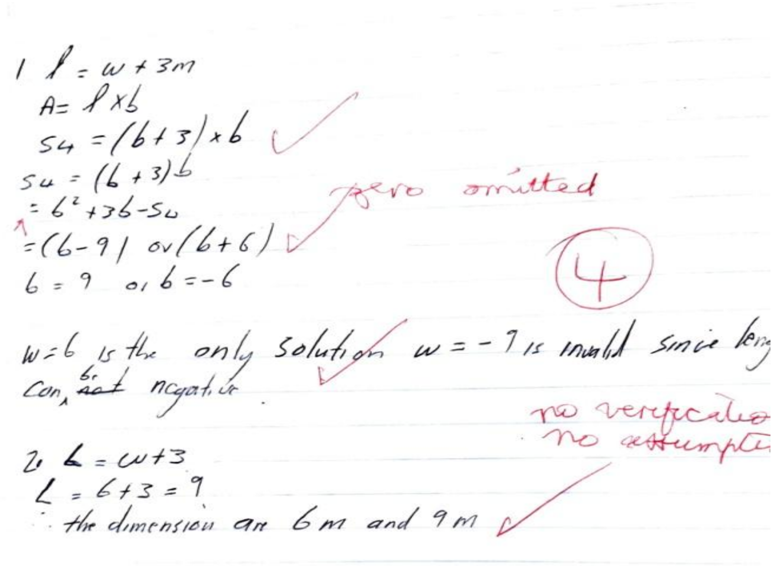


Figure 3: learner L46 response having a slip in solving

Figure 3 also denotes a learner's response to Question 1. Learner L46 successfully formulated the correct expression of the length in terms of breadth and eventually developed the correct quadratic equation. However, in terms of mathematical modelling processes, the learner could not make an assumption that would make the statements correct in all real-life contexts. They did not give an explanation why the solution -9 was invalid based on the given context. Lastly, there was no attempt to validate or verify the solutions. Since not all modelling processes were done, I concluded that the learner was partly competent in mathematical modelling. Regarding mathematical proficiencies, this learner could also be said to be partly proficient because they could not perform adaptive reasoning. The other three strands of mathematical proficiency (conceptual comprehension, fluency in procedures and strategic ability) were demonstrated correctly, except adaptive reasoning, which was to be achieved when the learner gave an explanation why -9 was an invalid solution and justified the solution. In this study, a learner would have been mathematical proficient, if they displayed capabilities in all the strands except the productive disposition, which could not be determined by the single test used to generate data for this study. Further analysis has been shown in Table 6.3.

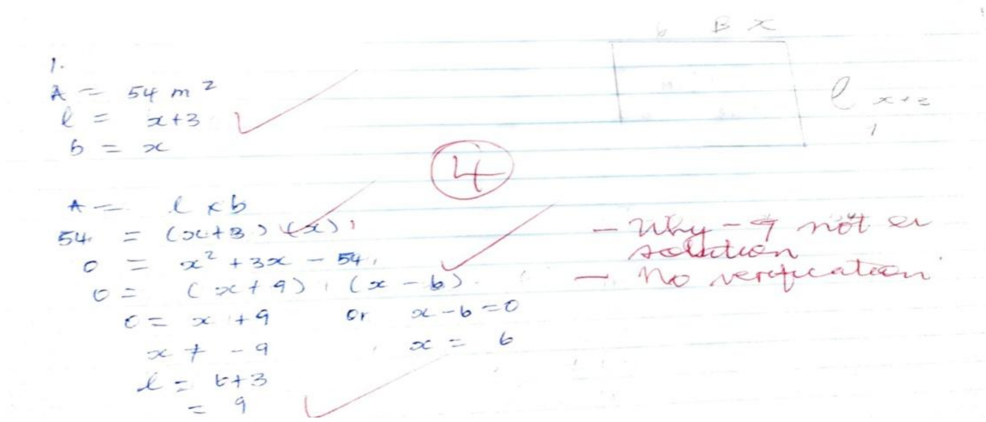


Figure 4*:* L25 response to Question 1

Figure 5 depicts learner L40's response to Question 1. Like learner L25 indicated in Figure 4, this learner was classified as partly competent in mathematical modelling and in mathematical proficient strands because of the same reasons ascribed to learner L25. Further analysis in terms of mathematical strands can be viewed in Table 1.

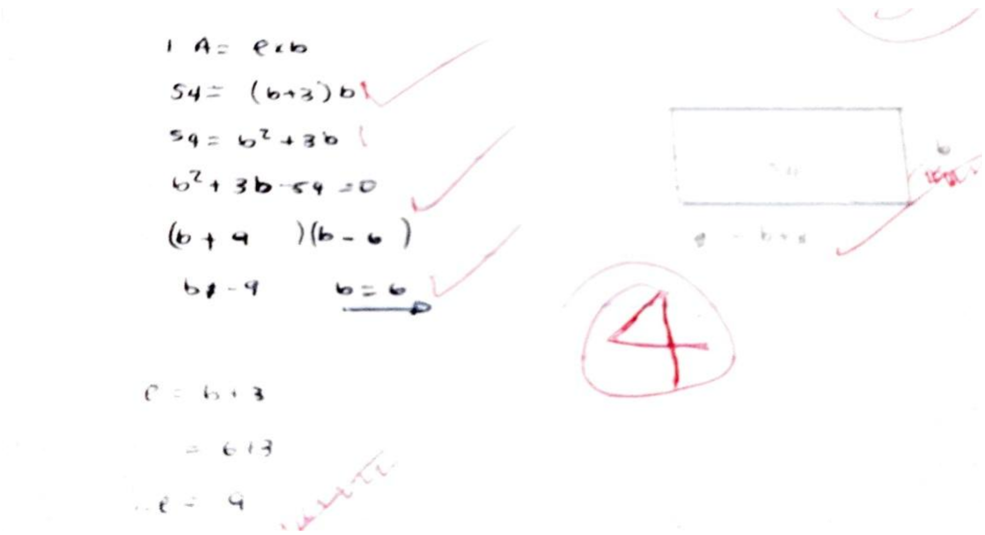


Figure 5: Learner L40's response to Question 1

The excerpt in Figure 6 displayed learner L67's response when resolving Question 1. The learner came out with the correct expression that showed the relations between length and breadth, developed the formula representing the area of the garden and proceeded to solve it correctly representing the area, solved it correctly. However, the learner never made any assumption and did not verify or validate the solutions. Since they could not make assumptions and verified the results, they were described as partly competent in mathematical modelling.

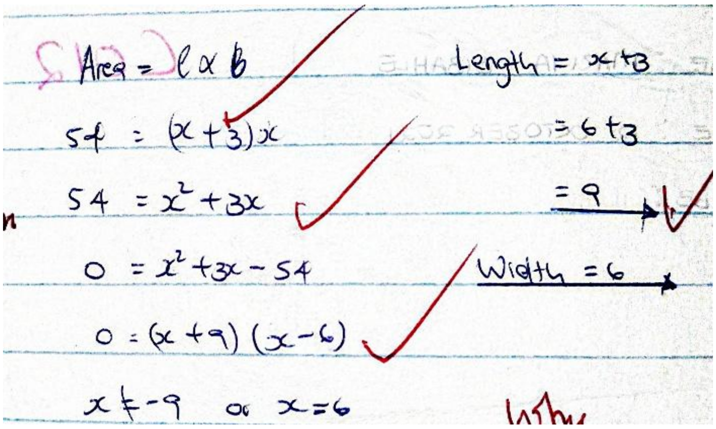


Figure 6: Learner L67's response to Question 1

From the extract, it can be seen that the learner was proficient in conceptual understanding (comprehended the problem very well), proficient in strategic competence (formulated the correct mathematical model) and proficient in procedural fluency (solved the mathematical model correctly). However, the learner could not verify the solutions and as a result, I identified this learner as partially or partly proficient in mathematical proficiency strands. In fact, all Despite failing to define variables or make assumptions, all learners were nevertheless able to create accurate mathematical models in this question. Making an accurate model showed that they had a thorough understanding of the problem.

From the word problems given in the test that was used to gather data for the study, learners needed to find out or identify the questions they had to answer. The next step was the assigning or allocating of variables (usually letters representing unknown numerical values in a mathematical equation or expression) in the problems. However, in this study, none of the participants stated what quantity each variable they have used represented. For example, in Question 1, we expected to see statements such as "Let l represents the length and b the breadth". Assigning of variable should have been followed by the formulation or development of a mathematical model which was either an equation or a mathematical expression comprising of these variables. After the formulation of equations, learners solved them to find the values of the variables. Most learners left their answer still equated to the variables. For instance, in Question 3, some learners they left their final answers as $x = 18$ and $y = 36$ instead of stating the answer to the problem. The expected answer to Question 3 would be any statement equivalent to this: "The dimensions that would create a garden of maximum area are 18 and 36 metres". The final step was supposed to be the verification or validating of the results which was not done by all participants.

The analysis of the results indicated that a majority of the participants were partial competent in mathematical modelling. This was also revealed by the rubric that none of the learners covered all the steps of modelling process. Even those who manage to make a correct formula and solve it correctly; they could not give or make correct assumption and checking the validity of their solutions. Some learners, though few, could not produce even a single correct step and therefore, I considered them as not competent in mathematical modelling. This result supports the findings of researchers Chan et al (2012) who also found that none of their participants were fully competent, just like in this study, participants were either not competent or partially competent. To be competent in mathematical modelling, a participant was expected to perform all the steps of the modelling process. Learners would write down variables without having assigned them to represent particular quantities. For example, if they wanted l to represent a length and w a width, they were supposed to state it clearly and say: "Let l represents the length and w the width" This would enable any one checking on their solution to understand what each of the written variable represented.

Another important step not done by all the participants was the making of assumptions. This result is consistent with studies by Seino (2005) and Krawitz et al. (2022), which found that learners found it difficult to create assumptions even though they are necessary for solving mathematical problems in real life. At the stage where they formulated the mathematical model, learners were to make their assumptions that would make it possible for them to resolve the problem. For example, in Question 1, to calculate the dimensions of a garden that would give an area that is $54m^2$. An assumption had to be made regarding the whole area where the garden was to be made. Just imagine if the area where the garden had to be made had a big rock or a permanent pond. The dimensions of 6m and 9 m were going to give usable area that would be less than $54m^2$. It is important to state the assumptions, giving what to be considered or neglected.

Some learners in Question 1 gave one of their final solutions as – 9 metres. This was also a forewarning or sign that they did not understand the context in which the problem was set up. At this level, one did not expect learners to give the length hat was negative. They were expected to know that length or distance is different from displacement, as it does not involve any direction and only focussing only on absolute value. The language in these questions could have been a barrier to some the learners, especially Question 3. Some, though not many, could not read the question carefully and with understanding. This was revealed by either their sketches or formulae. Question 3 demanded that learners calculated the lengths of the other three sides that could be covered by a fence of 72 metres, given that the fourth side was already covered by a rock. However, these learners considered all the four sides, which was an indication that they could not understand the question. All the participants in the three schools used English as the second language and isiZulu as the first language. Therefore, it was likely that some learners did not grasp what other questions entailed.

Failing by some learners to comprehend the word problems used in the test, especially Question 3 and 4 concurred with the studies conducted by Zerafa (2016), Stillman (2000) as cited in Leiß et al (2010), Leiß, Schukajlow, Blum, Messner and Pekrun (2010), Göksen-Zayim et al (2019), Govender and Machingura (2023) as well as English (2003). Zerafa (2016) investigated whether language determines or affects learners' performance in arithmetic word problems. The study found that language played an important role in solving word problems. Furthermore, the study revealed that learners were capable of solving even the most difficult word problems when the problems were in their first language, Maltese rather than in their second language, which is English. All these authors found that solving word problems was a challenge to many students because other than the challenges posed by Mathematics difficulties; learners need to be skillful in reading and interpreting language. Analysing learners' scripts was done to respond to the research question. As I analysed the data, I observed patterns and themes that emerged. In fact, some of these patterns could be observed during the marking of the test and during the analysis. The practice of methodically classifying extracts in a qualitative data to identify themes and patterns is known as qualitative coding. After making a thorough analysis, the following information was noted and categorised into themes shown in Table 1.

Table 1: Themes emerged from data analysis

Theme 1: Failure to make assumptions
• Never mentioned what variables they have used represent • No mentioning of conditions that would make their solutions to be always true
Theme 2: Incapability to obtain correct solutions
• Formulated incorrect mathematical model • Failed to work the solution fully (incomplete working) • Working carelessly ending up omitting essential information
Theme 3: Unfamiliarity with the verification of the results
• After obtaining solutions they did not validate them • Did not prove the correctness of their solutions with respect to the given situations
Theme 4: Inability to interpret the results
• Left their answers still equated to variables • Did not connect the solutions back to the problem situations • In Question 1 some learners did not explain why -9 was an invalid solution
Theme 5: Inability to understand given scenarios or English (Question 3)
• Read the scenario without understanding • Did not realise that although the garden is rectangular, only three sides had to be fenced
Theme 6: Failing to comprehend the given diagram(Question 4)
• Some learners could not consider the interior sides that divide the pasture into camps • Only the perimeter (sides right round the figure was considered).

CONCLUSION AND RECOMMENDATIONS

In this section, the findings of the study from an analysis of learners' scripts are outlined. I used tables to record what each learner has done regarding the understanding of the problem. This was assessed by checking whether a learner has made a relationship between variable and has made an assumption that would make the resolving of the problem to be true and easy. The next step was to check whether a learner has managed to translate the given situation into a mathematical model or equation and solved it correctly. The last two steps were to check whether a learner has interpreted their results, given a general statement explaining the solution, and verifying the obtained solutions. For a learner to be considered competent in mathematical modelling, they had to do all the five criteria correctly; otherwise, if one of the criteria had not been achieved, the learner was said to be partly competent in mathematical modelling. The criteria were drafted from mathematical modelling competencies mentioned by Maaß (2006) and Kaiser and Stender (2013) in Vorhölter et al (2019). Since this study used a qualitative approach, I had to observe the emerging patterns in the tables used to record data of each school. The analysis indicated that none of the participants from the three schools managed to carry out all the set criteria correctly and as a result, learners were classified as partly competent in mathematical modelling. This was based on the questions they have done or attempted.

None of the of the participants made or tried to make assumptions, interpreted the solutions in terms of the given problem or verified the solutions. my main concern was the failure to interpret the results. Results interpretation is crucial, particularly when word or contextualised problems deal with real-world scenarios. To make sure the mathematical solution makes sense in real-world situations, learners need to relate it back to the original context. Without the proper interpretation, the raw numerical or computational output is meaningless and cannot be correctly applied to the real-world situation it is intended to model.

- Learners must be taught this essential feature of mathematical modelling, which is the making of assumptions. These are all possible factors that may distort the worked solution, if not considered.

- Learners must be taught that once they come out with their own variables, they must specify what each variable stands for or represents. In most cases, when solving word problems, one has to develop an equation. Therefore, it is essential to specify what each variable represents in a real-life situation so that the known relationships can be expressed in equation form. For example, if in Question 1, it was necessary to specify the variables such as: let l represent length and b the breadth, it would be easy for anyone that knows the formula of a rectangle to express the area (A) in terms of l and b. Area of rectangle is then defined as $A = l \times b$

- Learners must be encouraged, where possible to come out with a sketch or diagram first before they produce a formula. Making and label a diagram is another step leading to the understanding of the problem.

- Learners must be taught to interpret their solutions. They must know that once they obtained their solutions, they have to interpret and translate them back into the context of the problem given. Most learners left their solutions still equated to variables.

- Teachers must insist to learners that must verify their solutions to check their validity. The ultimate step of the modelling processes is the verification or validation of the obtained solutions.

- Textbooks used in schools must be designed in a way that they must promote the teaching and learning of mathematical modelling processes. Thus, the questions and examples used must assist in the development of modelling processes.

- Tertiary institutions must offer short mathematical modelling courses to train the already practicing teachers and prospective teachers. Content to be covered should be relevant to the phase or grades the teacher is qualified to teach.

Teachers must expose learners to many word problems, which will provide learners more practice of reading with understanding. Learners must be trained on how to produce correct expressions or formulae on their own from the given word problems

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